

F/b -> 3

الفرقة الأولى

قسم مدني

فلويد

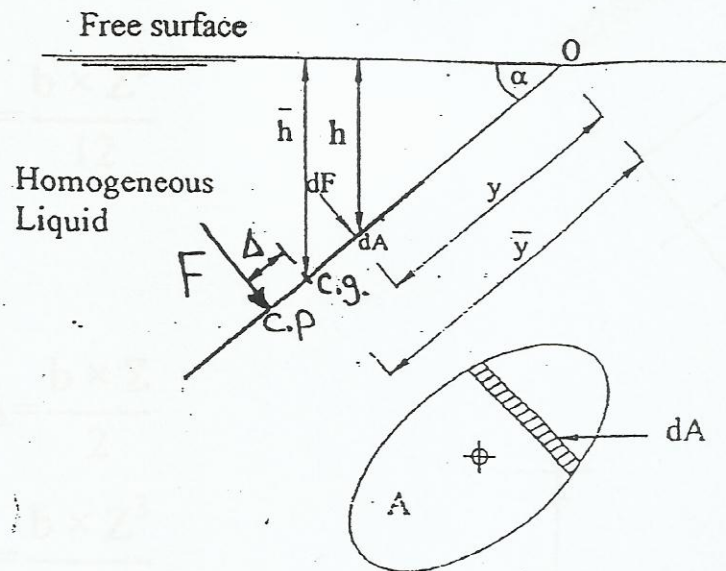
A-One

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3

HYDROSTATIC FORCES ON SUBMERGED PLANE SURFACES



- c.g. = center of gravity
 c.p. = center of pressure
 A = area of immersed surface (to the page)
 I = moment of inertia (about the axis to the page)
 h = **vertical distance** from the free surface to the c.g.
 y = **inclined distance** from the free surface to the c.g.
 Δ = inclined distance between (c.g) and (c.p)
 α = inclination angle (with the free surface)

➤ Rules:

- Hydrostatic Force

$$F = \gamma A \bar{h}$$

- Point of application

$$\Delta = \frac{I}{A \cdot \bar{y}}$$

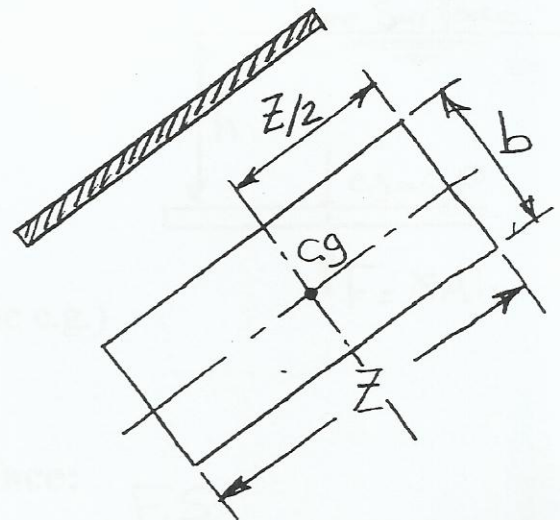
where: $\bar{y} = \frac{\bar{h}}{\sin \alpha}$

➤ Properties of Areas:

• Rectangle:

$$A = b \times Z$$

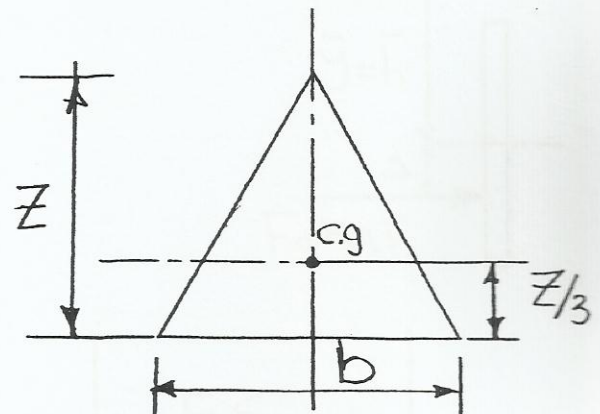
$$I = \frac{b \times Z^3}{12}$$



• Triangle:

$$A = \frac{b \times Z}{2}$$

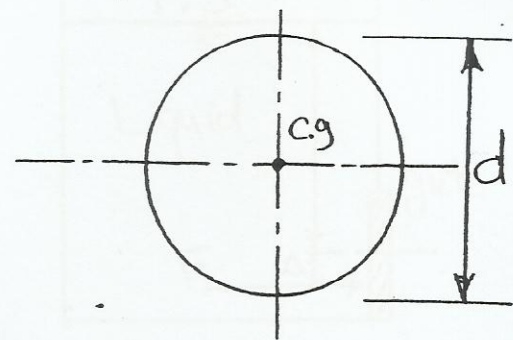
$$I = \frac{b \times Z^3}{36}$$



• Circle:

$$A = \frac{\pi \times d^2}{4}$$

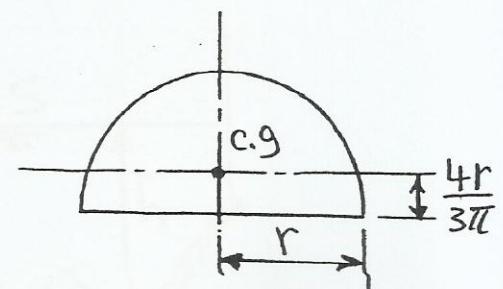
$$I = \frac{\pi \times d^4}{64}$$



• Semicircle:

$$A = \frac{\pi \times d^2}{8}$$

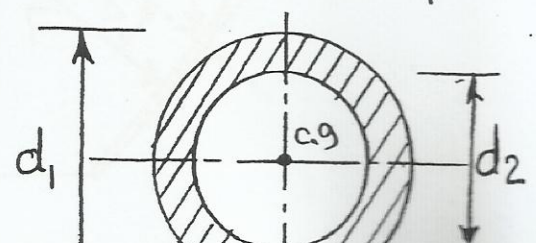
$$I = \frac{\pi \times d^4}{128} - \frac{d^4}{18\pi} \approx 0.11 r^2$$



• Hollow Circle

$$A = \frac{\pi}{4} (d_1^2 - d_2^2)$$

$$I = \frac{\pi}{64} (d_1^4 - d_2^4)$$



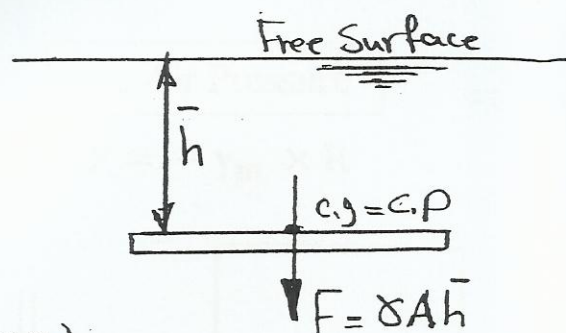
➤ Special Cases:

• Surfaces parallel to the free surface:

$$\alpha = \text{Zero} \rightarrow \bar{y} = \frac{\bar{h}}{\sin \alpha} = \infty$$

$$\therefore \Delta = \text{Zero}$$

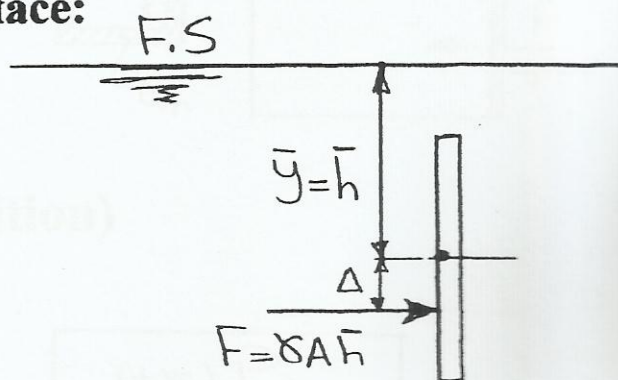
$$\rightarrow \boxed{\text{c.g.} = \text{c.p.}} \text{ (The force acts on the c.g.)}$$



• Surfaces perpendicular to the free surface:

$$\alpha = 90^\circ \rightarrow \sin \alpha = 1$$

$$\therefore \boxed{\bar{h} = \bar{y}}$$



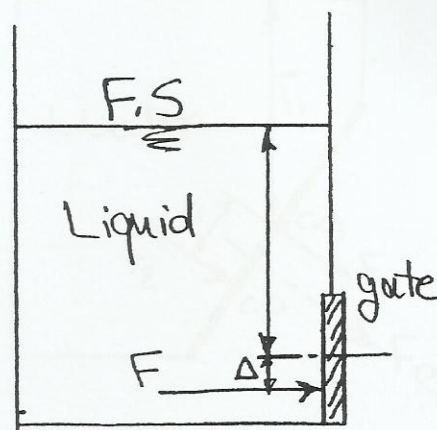
➤ Cases of Problems:

1) One Liquid:

Liquid surface is free ($P = 0$)

$$\rightarrow \bar{h} = \bar{y} \text{ (Vertical gate)}$$

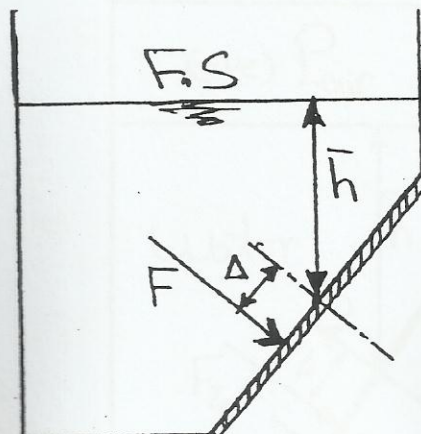
$$\therefore F = \gamma \times A \times \bar{h} \quad \Delta = \frac{I}{A \cdot \bar{y}}$$



For Inclined gate

$$\bar{y} = \frac{\bar{h}}{\sin \alpha}$$

$$\therefore F = \gamma \times A \times \bar{h} \quad \Delta = \frac{I}{A \cdot \bar{y}}$$

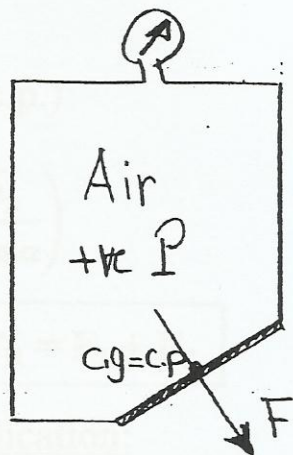


2) Air:

$$c.p = c.g$$

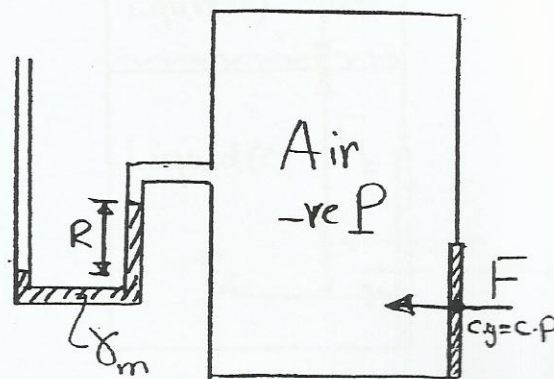
$$F = P \times A$$

+ve Air Pressure



-ve Air Pressure

$$P = -\gamma_m \times R$$



3) Liquid + Air: (Solving by Superposition)

Liquid & (+ve) Air pressure

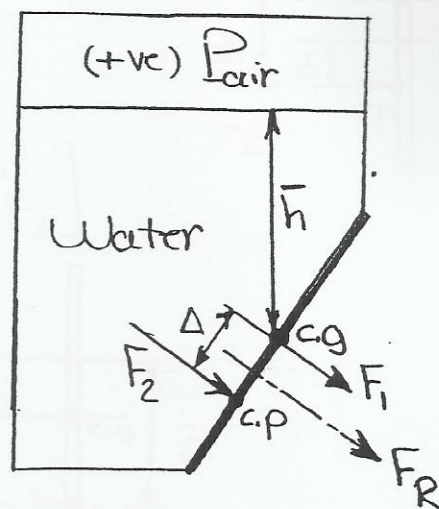
$$F_1 = P \times A \quad (\text{at c.g.})$$

$$F_2 = \gamma_w \times A \times \bar{h} \quad (\text{at c.p.})$$

$$\Delta = \frac{I}{A \cdot \bar{y}_2} \quad \left(\bar{y} = \frac{\bar{h}}{\sin \alpha} \right)$$

Resultant Force

$$F_R = F_1 + F_2$$



Liquid & (-ve) Air pressure

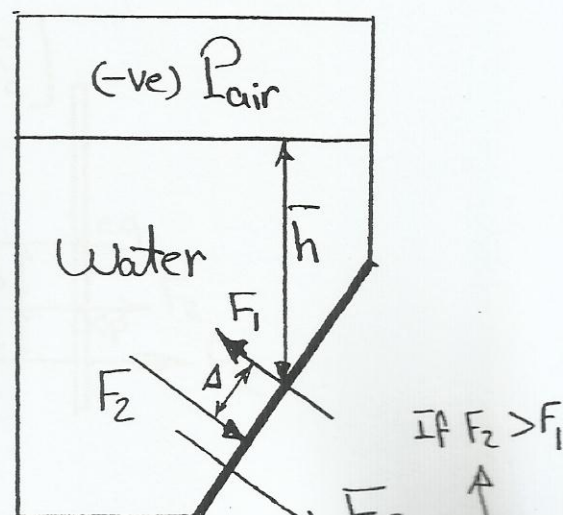
$$F_1 = P \times A \quad (\text{at c.g.})$$

$$F_2 = \gamma_w \times A \times \bar{h} \quad (\text{at c.p.})$$

$$\Delta = \frac{I}{A \cdot \bar{y}_2} \quad \left(\bar{y} = \frac{\bar{h}}{\sin \alpha} \right)$$

Resultant Force

$$F_R = F_1 - F_2$$



4) Two Liquids: (Solving by Superposition)

$$P = \gamma_1 \times h_1$$

$$F_1 = P \times A \text{ (at c.g.)}$$

$$F_2 = \gamma_2 \times A \times \bar{h}_2 \text{ (at c.p.)}$$

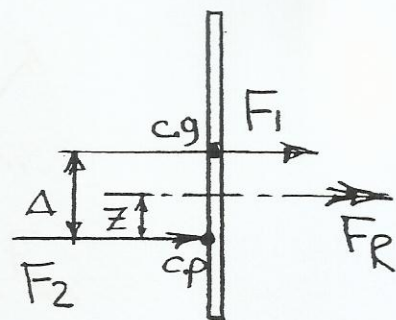
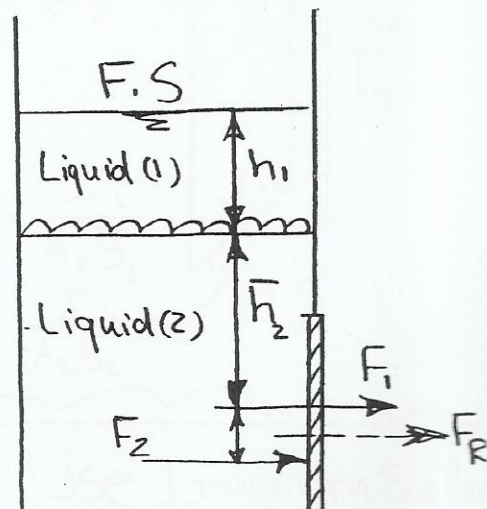
$$\Delta = \frac{I}{A \cdot \bar{y}_2} \quad \left(\bar{y}_2 = \frac{\bar{h}_2}{\sin \alpha} \right)$$

Resultant Force

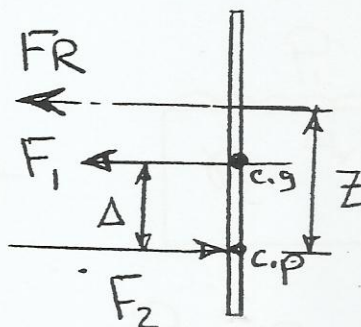
$$F_R = F_1 + F_2$$

To get (F_R) point of application:

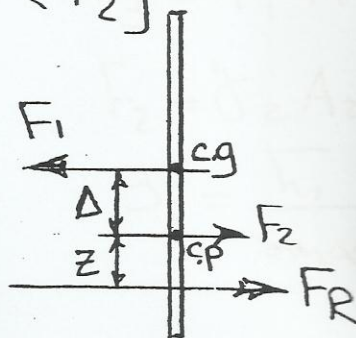
$$\sum M @ \text{c.p.} = \text{zero} \rightarrow F_1 \times \Delta = F_R \times Z \rightarrow \text{get } Z < \Delta$$



If F_1 is -ve (in case of -ve air pressure) $\rightarrow \begin{bmatrix} Z > \Delta \\ F_1 > F_2 \end{bmatrix}$

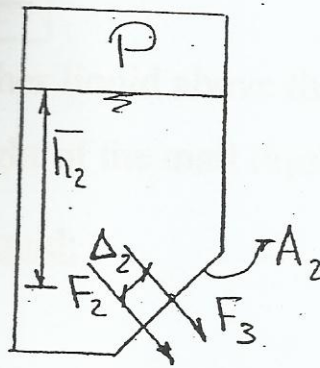
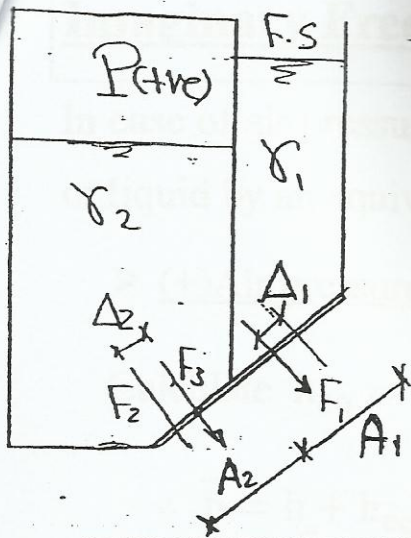


$\begin{bmatrix} F_1 < F_2 \end{bmatrix}$



5) Separated two liquids:

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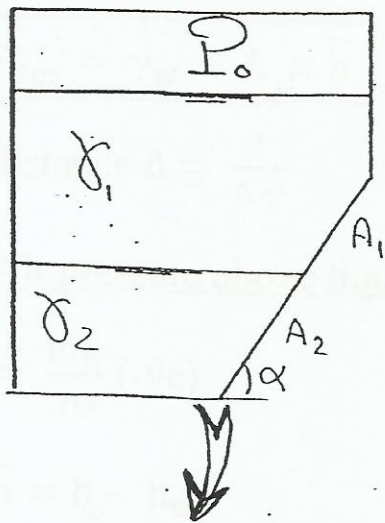
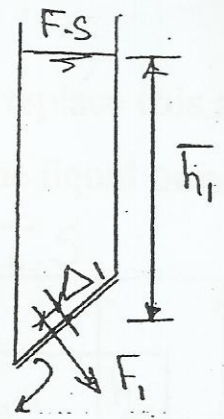
$$F_1 = \gamma A_1 \bar{h}_1$$

$$F_2 = \gamma A_2 \bar{h}_2$$

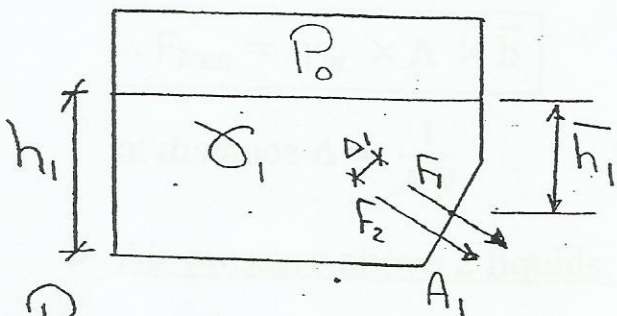
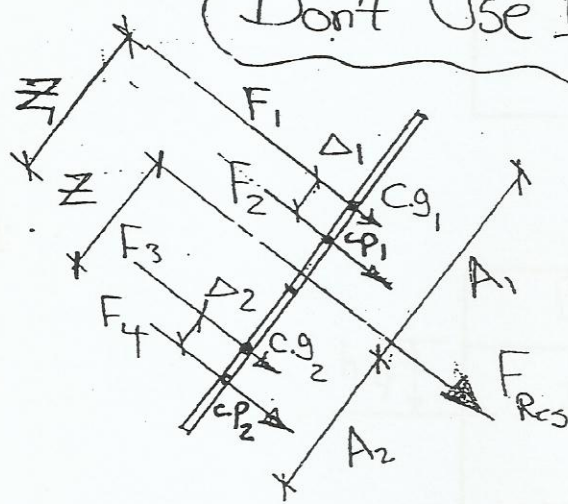
$$F_3 = P A_2$$

$$\Delta_1 = \frac{I_1}{A_1 \bar{y}_1}$$

$$\Delta_2 = \frac{I_2}{A_2 \bar{y}_2}$$



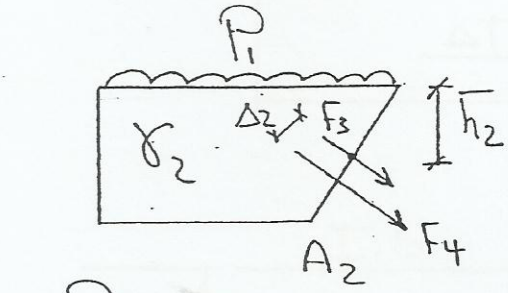
Don't Use Imaginary Surface



$$P_0 = \text{given} \rightarrow F_1 = P_0 A_1$$

$$F_2 = \gamma_1 A_1 \bar{h}_1 \quad \Delta_1 = \frac{I_1}{A_1 \bar{y}_1}$$

$$\bar{y}_1 = \frac{\bar{h}_1}{\sin \alpha}$$



$$P_1 = P_0 + \gamma_1 h_1$$

$$F_3 = P_1 A_2$$

$$F_2 = \gamma_2 A_2 \bar{h}_2 \quad \Delta_2 = \frac{I_2}{A_2 \bar{y}_2}$$

$$\bar{y}_2 = \frac{\bar{h}_2}{\sin \alpha}$$

$$\therefore F_{res} = F_1 + F_2 + F_3 + F_4$$

$$\text{To get } (\Sigma) \dots \Sigma M @ c_{p_2} = 0$$

Imaginary Free Surface

In case of air pressure or another liquid above the main liquid, we replace this air or liquid by an equivalent height of the main liquid and solve for one liquid only.

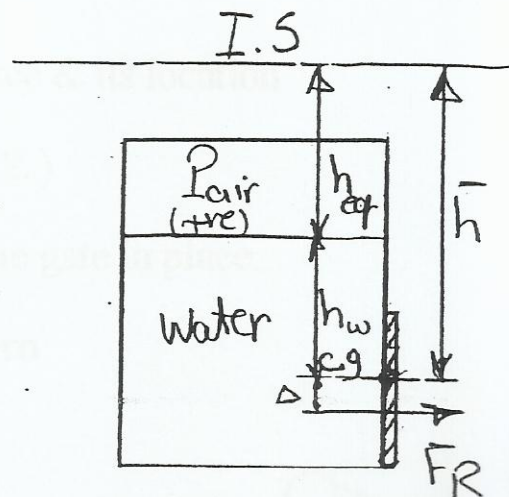
➤ (+) Air pressure above liquid:

Calculate $h_{eq} = \frac{P_{air}}{\gamma_w}$

$\therefore \bar{h} = h_w + h_{eq}$

$\therefore F_{Res} = \gamma_w \times A \times \bar{h}$

at distance $\Delta = \frac{I}{A \cdot \bar{y}}$



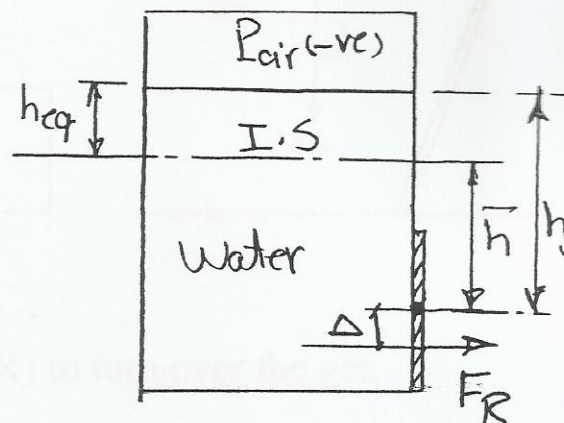
➤ (-) Air pressure above liquid:

$h_{eq} = \frac{P_{air}}{\gamma_w} (-ve)$

$\therefore \bar{h} = h_w - h_{eq}$

$\therefore F_{Res} = \gamma_w \times A \times \bar{h}$

at distance $\Delta = \frac{I}{A \cdot \bar{y}}$



➤ Air pressure above 2 liquids:

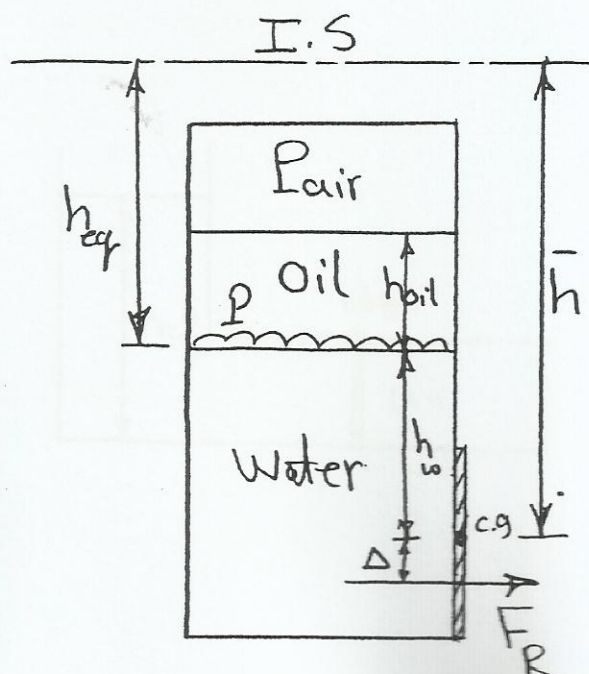
Calculate $P = P_{air} + \gamma_{oil} \times h_{oil}$

$\rightarrow h_{eq} = \frac{P}{\gamma_w}$

$\therefore \bar{h} = h_w + h_{eq}$

$\therefore F_{Res} = \gamma_w \times A \times \bar{h}$

at distance $\Delta = \frac{I}{A \cdot \bar{y}}$



What is required in problems?

1. Forces acting on surface (gate)

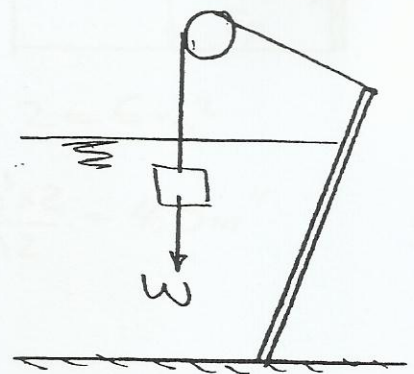
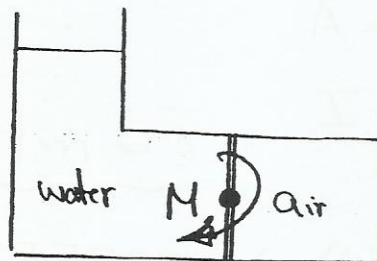
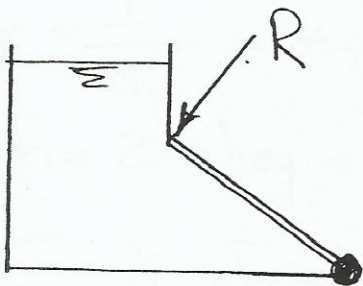
→ get (F) & no need for (Δ)

2. Force and line of action (OR) The resultant force & its location

→ get (F) & (Δ) → $F_{res} = \sum F$ & get (Z)

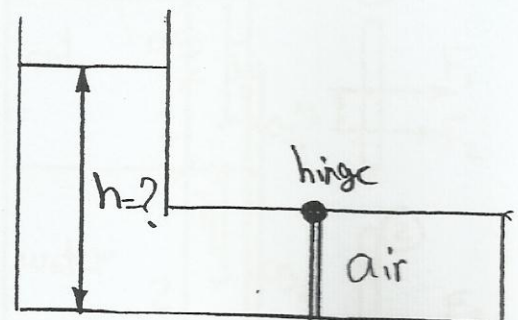
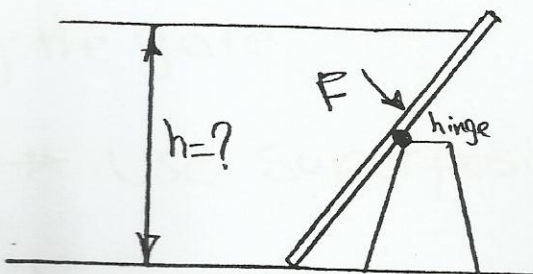
3. The force, moment or weight required to hold the gate in place.

→ get (F) & (Δ) → $\sum M @ \text{hinge} = \text{Zero}$



4. The liquid height required to hold the gate (OR) to turn over the gate.

→ $\sum M @ \text{hinge} = \text{Zero}$ → get (F) & (Δ) → get (h).



Get h to make the gate move

Ex.(1):

Find the resultant force and its point of application on the shown gate, giving that the air pressure is 30 kPa, for the following cases:

a. The water height is 3.5 m

b. The water height is 2 m

$$b_{\text{gate}} = 2 \text{ m}$$

Case (a) water surface is above the gate
 \Rightarrow Use imaginary surface

$$P = P_{\text{air}} + \gamma_o \cdot h_o$$

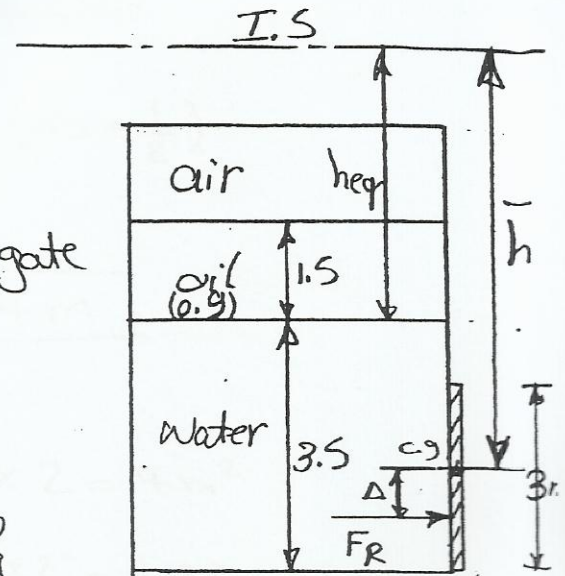
$$= 30 \times 10^3 + (0.9 \times 9810) \times 1.5 = 43243.5 \text{ Pa}$$

$$\therefore h_{\text{eq}} = \frac{P}{\gamma_w} = 4.41 \text{ m}$$

$$\hookrightarrow \bar{h} = 2 + h_{\text{eq}} = 6.41 \text{ m} = \bar{y}$$

$$\therefore F_R = \gamma_w \cdot \bar{h} \cdot A = 377293 \text{ N} \#$$

$$\text{at } \Delta = \frac{I}{A \bar{y}} = 0.117 \text{ m} \#$$



$$A = 3 \times 2 = 6 \text{ m}^2$$

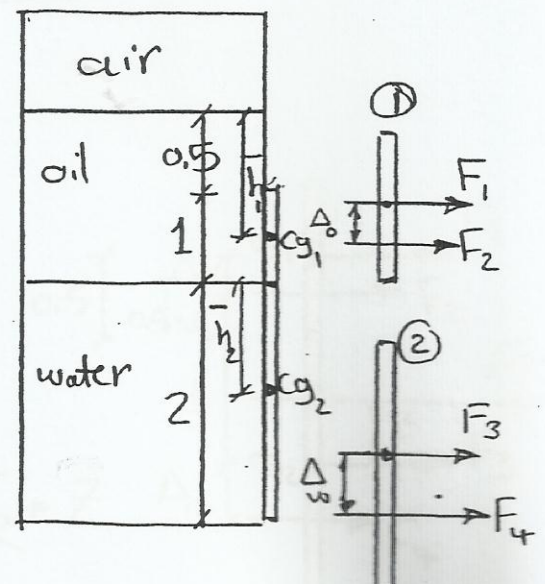
$$I = \frac{(3)^3 \times 2}{12} = 4.5 \text{ m}^4$$

Case (b)

water surface is below the top of the gate

\hookrightarrow Use superposition

& there will be 2 parts of the gate



* For part ① [oil] $A_1 = 1 \times 2 = 2 \text{ m}^2$

$$I_1 = \frac{(1)^3 \times 2}{12} = 0.167 \text{ m}^4$$

$$\xrightarrow{\text{air}} F_1 = P_{\text{air}} * A_1 = \boxed{60,000 \text{ N at C.g.①}}$$

$$\xrightarrow{\text{water}} F_2 = \gamma_o A_1 \bar{h}_1 = (0.9 * 9810) * 2 * (0.5 + \frac{1}{2})$$
$$= \boxed{17658 \text{ N}}$$

$$\text{at } \Delta_o = \frac{I_1}{A_1 \bar{y}_1} = \frac{0.167}{2 * 1} = \boxed{0.084 \text{ m}}$$

* For part ② [water] $A_2 = 2 \times 2 = 4 \text{ m}^2$

$$I_2 = \frac{(2)^3 \times 2}{12} = 1.33 \text{ m}^4$$

$$\xrightarrow{\text{oil+air}} P = P_{\text{air}} + \gamma_o h_o = 43243.5 \text{ Pa}$$

$$\therefore F_3 = P * A_2 = \boxed{172974 \text{ N}} \text{ at C.g.②}$$

$$\xrightarrow{\text{water}} F_4 = \gamma_w A_2 \bar{h}_2$$

$$= 9810 * 4 * \frac{2}{2} = \boxed{39240 \text{ N}}$$

$$\text{at } \Delta_w = \frac{I_2}{A_2 \bar{y}_2} = \frac{1.33}{4 * 1} = \boxed{0.333 \text{ m}}$$

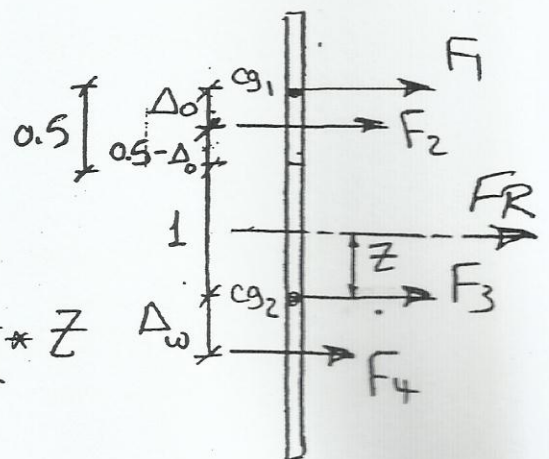
$$\therefore F_R = \sum F = \boxed{289872 \text{ N}}$$

To get point of application

$$\sum M @ \text{C.g.②} = 0$$

$$F_1 * 1.5 + F_2 (1.5 - \Delta_o) - F_4 * \Delta_w = F_R * Z$$

\hookrightarrow get (Z)



Ex.(2):

Find the moment (M) required to hold the gate closed in the shown figure.

* First we have to calculate the Pressure at the top of the water

$$P_a = P_b$$

$$5\delta_w \times 1 = P_c + \delta_w \times 7$$

$$\rightarrow P_c = -2\delta_w = -124.8 \text{ lb/ft}^2$$

Due to (-ve) Pressure above water

\rightarrow Use Super position

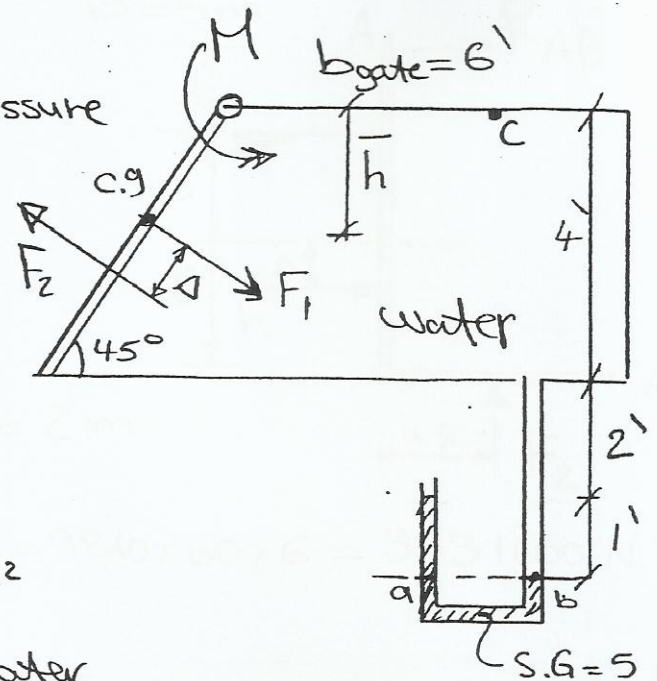
$$F_1 = P_c \times A = 124.8 \left(\frac{4}{\sin 45} \times 6 \right) = 4236 \text{ lb at c.g}$$

$$F_2 = \gamma A \bar{h} = 62.4 \left(\frac{4}{\sin 45} \times 6 \right) \times 2 = 4236 \text{ lb}$$

$$\text{at } \Delta = \frac{I}{A \bar{y}} = \frac{90.5}{33.9 \times \frac{2}{\sin 45}} = 0.943'$$

$$\therefore M = F_2 \times \left(\frac{2}{\sin 45} + \Delta \right) - F_1 \times \left(\frac{2}{\sin 45} \right)$$

$$= \boxed{3993 \text{ lb.ft}}$$

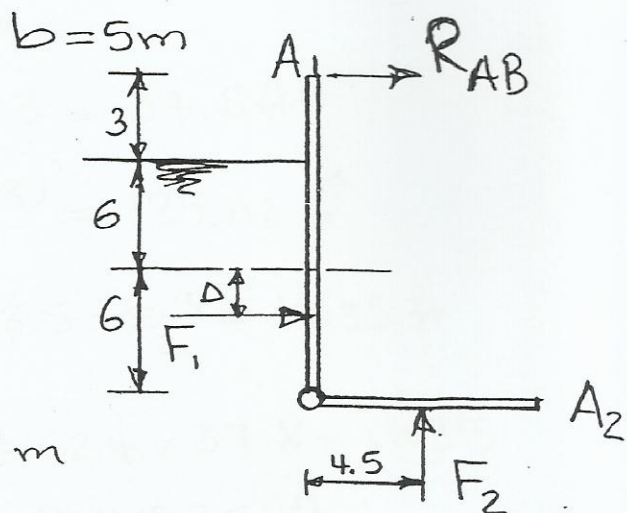
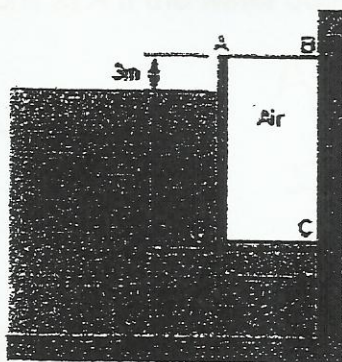


✓ 3/1-21 Gate AOC shown is 5m wide and is hinged along O (i.e., it can rotate freely about axis o). Neglecting the weight of the gate, determine the force in bar AB.

Req: -

R_{AB}

Solution:



$$A_1 = 12 \times 5 = 60 \text{ m}^2$$

$$I_1 = 5(12)^3/12 = 720 \text{ m}^4$$

$$\bar{h} = \bar{y} = 6 \text{ m}$$

$$\Delta = \frac{I}{A\bar{y}} = 2 \text{ m}$$

$$F_1 = \gamma A_1 \bar{h}_1 = 9810 \times 60 \times 6 = 3531600 \text{ N}$$

$$A_2 = 9 \times 5 = 45 \text{ m}^2$$

$$I_2 = 5(9)^3/12 = 303.75 \text{ m}^4$$

$$\bar{h}_2 = 12$$

$$F_2 = \gamma A_2 \bar{h}_2$$

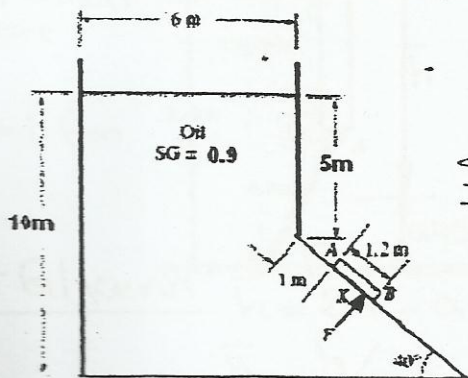
$$= 9810 \times 45 \times 12 = 5297400 \text{ N}$$

$$\Delta_2 = 0 \text{ (} F_2 \text{ at center)}$$

$$\sum M @ \text{hinge} = \text{Zero} \Rightarrow R_{AB} \times 15 + F_1(6 - \Delta) = F_2 \times 4.5$$

$$\Rightarrow R_{AB} = 647460 \text{ N}$$

✓ 3/1-22 Gate AB in Fig. is 1.2 m long and 1.8 m into the paper. Neglecting atmospheric pressure, compute the force F on the gate and its center-of-pressure position X .

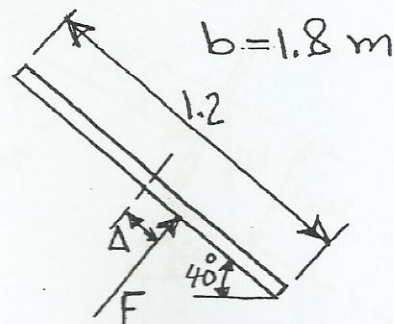


Req: F & Δ

Sol: -

$$A = 1.2 \times 1.8 = 2.16 \text{ m}^2$$

$$I = \frac{(1.2)^3 \times 1.8}{12} = 0.259 \text{ m}^4$$



$$\bar{h} = 5 + 1.6 \sin 40 = 6.03 \text{ m} \quad \bar{y} = \bar{h} / \sin 40 = 9.38 \text{ m}$$

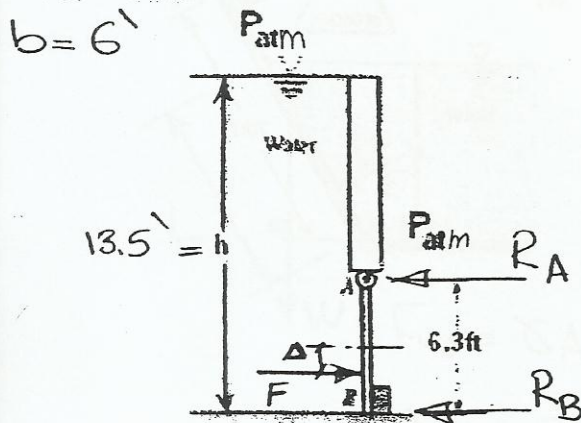
$$F = \gamma_{oil} A \bar{h} = (9810 \times 0.9) \times 2.16 \times 6.03 = 114995.95 \text{ N}$$

$$\Delta = \frac{I}{A\bar{y}} = 0.013 \text{ m}$$

3/1-23 Gate AB in Fig. is 6 ft wide into the paper, hinged at A, and restrained by a stop at B. The water is at 25°C. Compute

(a) The force on stop B

(b) The reactions at A if the water depth $h = 13.5$ ft.



$$A = 6 \times 6.3 = 37.8 \text{ ft}^2$$

$$I = \frac{6 (6.3)^3}{12} = 125.02 \text{ ft}^4$$

$$\bar{h} = \bar{y} = 13.5 - \frac{6.3}{2} = 10.35 \text{ ft}$$

$$F = \gamma A \bar{h} = 62.4 \times 37.8 \times 10.35 = 24412.75 \text{ lb}$$

$$\Delta = \frac{I}{A \bar{y}} = 0.32 \text{ ft}$$

$$\sum M @ \text{hinge} = \text{Zero}$$

$$F \left(\frac{6.3}{2} + \Delta \right) = R_B \times 6.3 \Rightarrow R_B = 13446.4 \text{ lb}$$

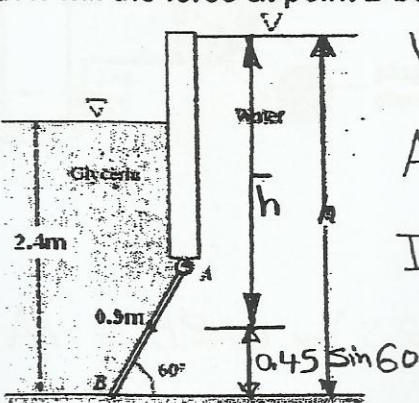
$$\sum F_x = 0 \Rightarrow R_A + R_B = F \Rightarrow R_A = 10966.4 \text{ lb}$$

3/1-24 Gate AB in Fig. is a homogeneous mass of 280 kg, 1.6 m wide into the paper, hinged at A, and resting on a smooth bottom at B. All fluids are at 26°C. For what water depth h will the force at point B be zero?

From tables

$\gamma_{\text{gly}} = 1.26$
glycerine

$b = 1.6 \text{ m}$



$$W_{\text{gate}} = m \times g = 280 \times 9.81 = 2746.8 \text{ N}$$

$$A = 0.9 \times 1.6 = 1.44 \text{ m}^2$$

$$I = \frac{(0.9)^3 \times 1.6}{12} = 0.097 \text{ m}^4$$

For Glycerin $\bar{h} = 2.4 - 0.45 \sin 60 = 2.01 \text{ m}$

$$\bar{y} = \bar{h} / \sin 60 = 2.32 \text{ m}$$

$$F_g = \gamma_g A \bar{h} = (1.26 \times 9810) \times 1.44 \times 2.01 = 35776.5 \text{ N}$$

$$\Delta_g = \frac{I}{A \bar{y}} = 0.029 \text{ m}$$

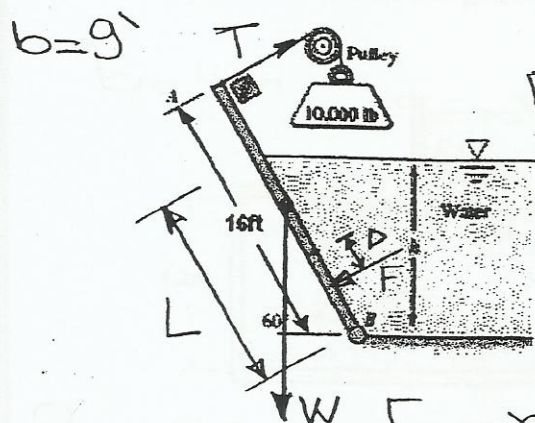
For Water

$$F_w = \gamma_w A \bar{h} = 14126.4 \bar{h} \quad \Delta = \frac{I}{A \bar{y}} = \frac{0.058}{\bar{h}}$$

$$\sum M @ \text{hinge} = F_g (0.45 + \Delta_g) - F_w (0.45 + \Delta_w) + W (0.45 \cos 60^\circ) = 0$$

$$\bar{h} = 2.66 \text{ m}$$

3/1-25 Gate AB in Fig. is 15 ft long and 9 ft wide into the paper and is hinged at B with a stop at A. The water is at 12° C. The gate is 0.9-in-thick steel, SG = 7.85. Compute the water level h for which the gate will start to fall.



$$T = 10,000 \text{ lb}$$

$$\text{For gate } W = V \cdot \gamma_{st} = (16 \cdot 9 \cdot \frac{0.9}{12}) \cdot 7.85 \cdot 62.4$$

$$= 5290.27 \text{ lb}$$

$$A = (9L) \text{ ft}^2 \quad I = \frac{9L^3}{12} = (0.75L^3) \text{ ft}^4$$

$$\bar{h} = \frac{h}{2} \quad \bar{y} = \frac{\bar{h}}{\sin 60} = \frac{h}{2 \sin 60} \cdot L = \frac{h}{\sin 60}$$

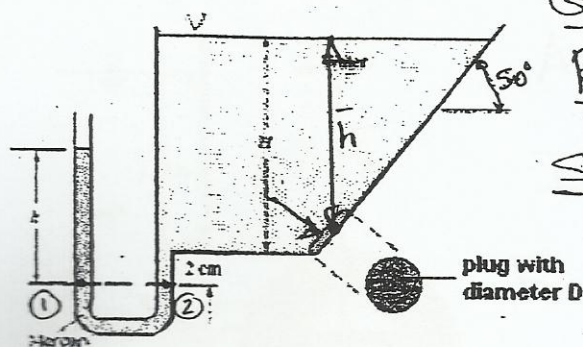
$$F_w = \gamma A \bar{h} = 62.4 \cdot 9L \cdot \frac{h}{2} = 324.24h$$

$$\Delta = \frac{I}{A \bar{y}} = \frac{0.75L^3}{9L \cdot \frac{L}{2}} = 0.167L = 0.192h$$

$$\sum M @ B = 0$$

$$T \cdot 16 = W \cdot 8 \cos 60 + F \left(\frac{h}{2 \sin 60} - \Delta \right) \rightarrow \boxed{h = 33.33 \text{ m}}$$

3/1-26 The tank in Fig. has a 6-cm-diameter plug at the bottom on the right. All fluids are at 20° C. The plug will pop out if the hydrostatic force on it is 45 N. For this condition, what will be the reading h on the mercury manometer on the left side?



$$\text{Given } D = 6 \text{ cm} \quad F = 45 \text{ N}$$

$$\text{Req. Manometer reading } (h)$$

$$\text{Sol. } A = \frac{\pi D^2}{4} = 2.83 \times 10^{-3}$$

$$I = \frac{\pi D^4}{64} = 6.36 \times 10^{-7}$$

$$\bar{y} = \frac{\bar{h}}{\sin 50}$$

$$F = \gamma A \bar{h} = 9810 \cdot (2.83 \times 10^{-3}) \cdot \bar{h} = 27.76 \bar{h} = 45$$

$$\rightarrow \bar{h} = 1.62 \rightarrow H = \bar{h} + 0.03 \sin 50 = 1.644 \text{ m}$$

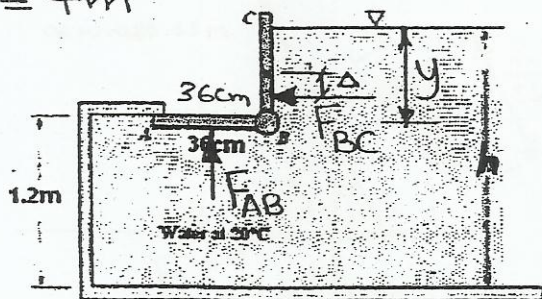
In manometer:

$$P_1 = P_2$$

$$\gamma \cdot h = \gamma_w \cdot (H + 0.02) \rightarrow \boxed{h = 0.122 \text{ m}}$$

- ✓ 3/1-27 Gate ABC in Fig. has a fixed hinge line at B and is 4 m wide into the paper. The gate will open at A to release water if the water depth is high enough. Compute the depth h for which the gate will begin to open.

$$b = 4 \text{ m}$$



Part AB

$$A = 0.36 \times 4 = 1.44 \text{ m}^2$$

$$I = \frac{(0.36)^3 \times 4}{12} = 0.016 \text{ m}^4$$

$$\bar{h} = \bar{y} = y$$

Part BC

$$A = 4y \quad I = \frac{4y^3}{12} = 0.33y^3$$

$$\bar{h} = \bar{y} = y/2$$

$$F_{BC} = \gamma A \bar{h} = 2\gamma y^2$$

$$\Delta = \frac{I}{A\bar{y}} = y/6$$

$$F_{AB} = \gamma_w A \bar{h} = 1.44 \gamma_w y \text{ at C.g.}$$

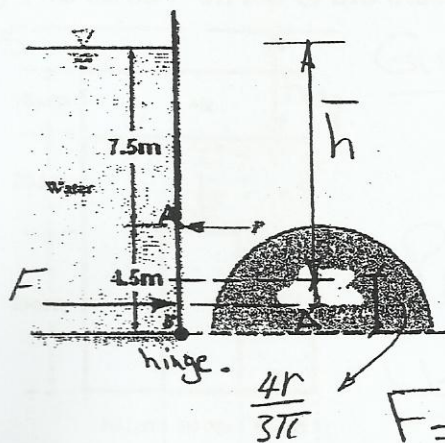
$$\sum M @ B = 0$$

$$F_{AB} \times 0.18 = F_{BC} \left(\frac{y}{2} - \Delta \right)$$

$$1.44 \gamma_w y \times 0.18 = 2 \gamma_w y^2 \left(\frac{y}{2} - \frac{y}{6} \right)$$

$$\rightarrow y = 0.624 \rightarrow h = y + 1.2 = 1.824 \text{ m}$$

- 3/1-28 Gate AB in Fig. is semicircular, hinged at B, and held by a horizontal force P at A. What force P is required for equilibrium $T = 12^\circ \text{ C}$?



$$A = \frac{\pi (4.5)^2}{2} = 31.81 \text{ m}^2$$

$$I = 0.11 (4.5)^4 = 45.11 \text{ m}^4$$

$$\bar{h} = \bar{y} = 12 - \frac{4(4.5)}{3\pi} = 10.09 \text{ m}$$

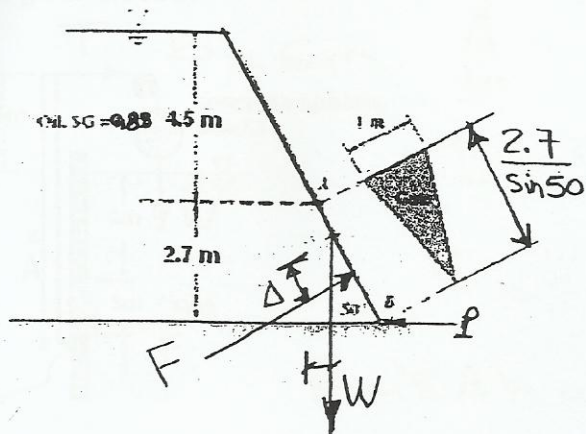
$$F = \gamma A \bar{h} = 3148690 \text{ N}$$

$$\Delta = \frac{I}{A\bar{y}} = 0.141 \text{ m}$$

$$\sum M @ B = 0$$

$$P \times 4.5 - F \left(\frac{4(4.5)}{3\pi} - \Delta \right) = 0 \rightarrow P = 1237687 \text{ N}$$

3/1-29 Isosceles triangle gate AB in Fig. is hinged at A and weighs 2350 N. What horizontal force P is required at point B for equilibrium?



$$A = \frac{2.7}{\sin 50} \times 1 \times 0.5 = 1.76 \text{ m}^2$$

$$I = \frac{1}{36} \left(\frac{2.7}{\sin 50} \right)^3 = 1.22 \text{ m}^4$$

$$\bar{h} = 4.5 + \frac{2.7}{3} = 5.4 \text{ m}$$

$$\bar{y} = \frac{\bar{h}}{\sin 50} = 7.05 \text{ m}$$

$$F = \gamma A \bar{h} = (0.83 \times 9810) \times 1.76 \times 5.4 = 77384.4 \text{ N}$$

$$\Delta = \frac{I}{A \bar{y}} = 0.098 \text{ m}$$

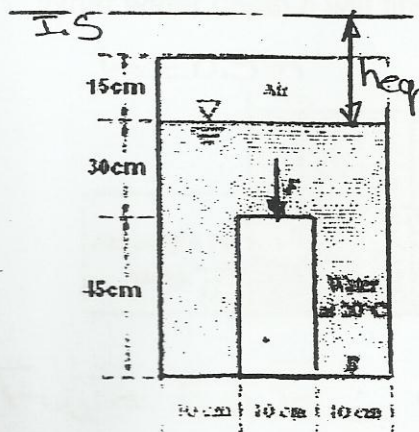
$$P = 35824 \text{ N}$$

$$\sum M @ A = 0 \rightarrow P \times 2.7 + W \left(\frac{2.7}{3} \tan 50 \right) - F \left(\frac{2.7}{3 \sin 50} + \Delta \right) = 0$$

3/1-30 The cylindrical tank in Fig. has a 45-cm-high cylindrical insert in the bottom. The pressure at point B is 265 kPa. Find:

(a) The pressure in the air space?

(b) The force F on top of the insert? (Neglect air pressure outside the tank)



Given $P_B = 265 \text{ kPa}$

$$A = \frac{(0.1)^2 \pi}{4} = 7.85 \times 10^{-3} \text{ m}^2$$

$$P_{\text{Air}} = P_B - \gamma_w (0.75) = 257643 \text{ Pa}$$

Using Superposition

$$F_{\text{air}} = P_{\text{air}} \times A = 2022.5 \text{ N}$$

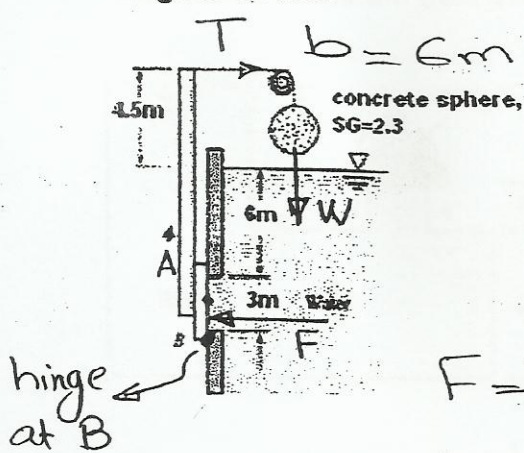
$$F_{\text{water}} = \gamma A \bar{h} = 9810 \times (7.85 \times 10^{-3}) \times 0.3 = 23.1 \text{ N}$$

$$\rightarrow F_{\text{Total}} = F_{\text{air}} + F_{\text{water}} = 2045.6 \text{ N}$$

Using Imaginary Surface $h_{\text{eq}} = \frac{P_{\text{air}}}{\gamma_w} = 26.26 \text{ m}$

$$\rightarrow \bar{h} = h_{\text{eq}} + 0.3 = 26.56 \rightarrow F_{\text{Total}} = \gamma A \bar{h} = 2045.6 \text{ N}$$

3/1-31 In Fig. gate AB is 6 m wide into the paper and is connected by a rod and pulley to a concrete sphere (SG = 2.3). What diameter of the sphere is just sufficient to keep the gate closed?



$$A_{\text{gate}} = 3 \times 6 = 18 \text{ m}^2$$

$$I = \frac{(3)^3 \times 6}{12} = 13.5 \text{ m}^4$$

$$\bar{h} = \bar{y} = 6 + \frac{3}{2} = 7.5 \text{ m}$$

$$F = \gamma A \bar{h} = 1324350 \text{ N}$$

$$\Delta = \frac{I}{A \bar{y}} = 0.1 \text{ m}$$

$$*V = \frac{4}{3} \pi r^3$$

$$r = 1.13 \text{ m}$$

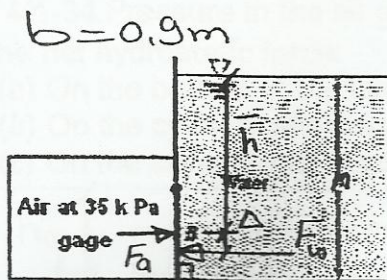
$$\therefore \textcircled{D} = 2.26 \text{ m}$$

$$\sum M @ B = 0 \rightarrow T \times 13.5 = F \times (1.5 - \Delta)$$

$$\rightarrow T = 137340$$

$$\text{But } W = T = \gamma_{\text{conc}} \times V \rightarrow V = \frac{137340}{2.3 \times 9810} = 6.09 \text{ m}^3$$

✓ 3/32 Gate B in Fig. is 60 cm high, 90 cm wide into the paper, and hinged at the top. What water depth h will first cause the gate to open?



$$A_{\text{gate}} = 0.6 \times 0.9 = 0.54 \text{ m}^2$$

$$I = \frac{(0.6)^3 \times 0.9}{12} = 0.0162 \text{ m}^4$$

$$\bar{h} = \bar{y}$$

$$F_w = \gamma A \bar{h} = 5297.4 \bar{h}$$

$$\Delta = \frac{I}{A \bar{y}} = \frac{0.03}{\bar{h}}$$

$$F_{\text{air}} = P_{\text{air}} \times A$$

$$= 35 \times 10^3 \times 0.54$$

$$F_{\text{air}} = 64814.8 \text{ N at cg}$$

When the gate starts to open

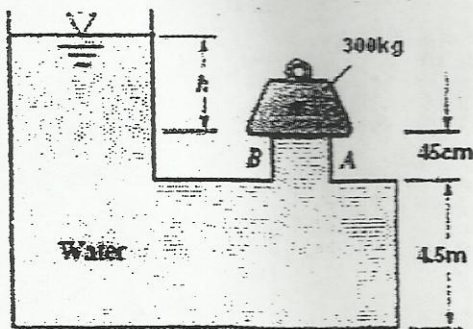
$$\sum M @ \text{hinge} = 0$$

$$\bar{h} = 12.14 \text{ m}$$

$$F_{\text{air}} \times 0.3 = F_w (0.3 + \Delta)$$

$$19444.4 = 5297.4 \bar{h} (0.3 + \frac{0.03}{\bar{h}}) \rightarrow \therefore h = \bar{h} + 0.3 = 12.44 \text{ m}$$

3/1-33 In Fig. The cover gate AB closes a circular opening 60 cm in diameter. The gate is held closed by a 300-kg mass as shown. Assume standard gravity at 27° C. At what water level h will the gate be dislodged? Neglect the weight of the gate.



$$A = \frac{\pi(0.6)^2}{4} = 0.283 \text{ m}^2$$

$$F_w = W = mg$$

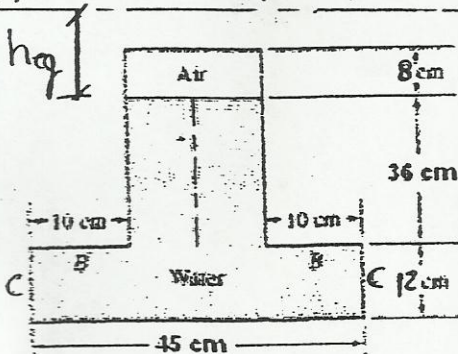
$$\rho_w Ah = m \cdot g$$

$$\rightarrow h = \frac{300 \cdot 9.81}{0.283 \cdot 9810} = \boxed{1.06 \text{ m}}$$

✓ 4/1-34 Pressure in the air gap in Fig. is 11 k Pa gage. The tank is cylindrical. Calculate the net hydrostatic force:

- On the bottom of the tank
- On the cylindrical sidewall CC
- On the annular plane panel BB.

$$h_{eq} = \frac{P_{air}}{\rho_w} = 1.12 \text{ m}$$



(a) Force on bottom

$$A = \frac{\pi(0.45)^2}{4} = 0.159 \text{ m}^2$$

$$\bar{h} = h_{eq} + 0.48 = 1.6 \text{ m}$$

$$F_{\text{Bottom}} = \rho A \bar{h} = 2495.7 \text{ N}$$

(b) On sidewall

$$A = 0.45 \times 0.12 = 0.054 \text{ m}^2$$

$$\bar{h} = h_{eq} + 0.36 + 0.06 = 1.54 \text{ m}$$

$$F = 815.8 \text{ N}$$

S.W

(c) On plane BB

$$A = \frac{\pi}{4} (0.45^2 - 0.25^2) = 0.11 \text{ m}^2$$

$$\bar{h} = h_{eq} + 0.36 = 1.48 \text{ m}$$

$$F_{\text{BB}} = 1597 \text{ N}$$