

\* section: Eng: Rami Barouk.

النهايات و النهايات

$$\lim_{(x,y) \rightarrow (x_0, y_0)} P(x,y) = P(x_0, y_0)$$

$$x = X + x_0$$

$$y = Y + y_0$$

نستخدم

$$\lim_{(x,y) \rightarrow (x_0, y_0)}$$

$$\Rightarrow \lim$$

$$(x,y) \rightarrow (0,0)$$

نستخدم المسارات  $y=mx$  و  $y=mx^2$  ونجرب النهاية  $(y=mx, y=mx^2)$  ونجرب النهاية  $(y=mx, y=mx^2)$

$$x = r \cos \theta, y = r \sin \theta, \lim_{r \rightarrow 0}$$

معد

نستخدم المسارات  $y=mx$  و  $y=mx^2$  ونجرب النهاية  $(y=mx, y=mx^2)$

$$5) \lim$$

$$(x,y) \rightarrow (\pi/2, 1) \quad \frac{3y+1+\cos x}{y^2+2\sin x} = \frac{4}{5}$$

$$6) \lim$$

$$(x,y) \rightarrow (0,0)$$

$$\frac{x^2 y}{x^4 + y^2} = \frac{0}{0}$$

نستخدم المسار  $y=mx^2$

$$\lim_{x \rightarrow 0} \frac{mx^4}{x^4 + m^2 x^4} = m$$

$$1+m^2$$

depends on m, so Not exist.

$$7) \lim$$

$$(x,y) \rightarrow (0,0)$$

$$\frac{xy^2}{x^2 + y^2}$$

we use Polar Coordinates.

$$\lim_{r \rightarrow 0}$$

$$\frac{r^3 \cos \theta \sin^2 \theta}{r^2} = 0$$

so limit is exist.

$$* \lim$$

$$(x,y) \rightarrow (0,0)$$

$$\frac{xy}{(x^2 + y^2)^{3/2}} = \frac{0}{0}$$

$$y = x^2$$

$$x^3 + y^3$$

$$y^2 = x$$

$$xy = x^3$$

$$y = \sqrt{x}$$

$$y = x^2$$



∴ we use  $y = m\sqrt{x}$  or  $y = mx^2$ .

$$\lim_{x \rightarrow 0} \frac{m x \sqrt{x}}{(x^2 + m^2 x)^{3/2}}, \quad \lim_{x \rightarrow 0} \frac{m}{(x + m^2)^{3/2}} = \frac{1}{m^2} \text{ Not exist bec. it depends on } m.$$

\* Example:

$$F(x, y) = \frac{\tan xy^2}{x^3 + y^3} \quad (x, y) \neq (0, 0)$$

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{\tan xy^2}{x^3 + y^3} = \text{value of } K \quad (x, y) = (0, 0)$$

$$* \lim_{(x, y) \rightarrow (0, 0)} \frac{\tan(xy^2)}{x^3 + y^3} \cdot xy^2$$

it depends on The value of  $m$  so it doesn't exist

$$= \lim_{(x, y) \rightarrow (0, 0)} \frac{xy^2}{x^3 + y^3}, \quad \text{we use } y = mx = \frac{m^2}{1+m^5}$$

\* limit isn't exist so The function isn't Continuous for all values of  $K$ .

$$xy^2 = x^3$$

$$y^2 = x^2$$

$$x = y$$